Braced Steel Column Verification Example

AISC Design Examples

AISC 13th Edition

Example H.1b W-shape Column Subjected to Combined Compression and Bending Moment About Both Axes (braced frame)

Verify if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed below, obtained from a second order analysis that includes second-order effects. The unbraced length is 14 ft and the member has pinned ends. $KL_x = KL_y = L_b = 14.0$ ft

LRFD	ASD
$P_u = 400$ kips $M_{ux} = 250$ kip-ft $M_{uy} = 80.0$ kip-ft	$P_a = 267$ kips $M_{ax} = 167$ kip-ft $M_{ay} = 53.3$ kip-ft

Solution:

Material Properties: Manual Table 2-3

ASTM A992 $F_y = 50$ ksi $F_u = 65$ ksi

Take the available axial and flexural strengths from the Manual Tables

LRFD	ASD	
at $KL_y = 14.0$ ft, $P_c = \phi_c P_n = 1130$ kips	at $KL_y = 14.0$ ft, $P_c = P_n/\Omega_c = 751$ kips	Manual Table 4-1
at $L_b = 14.0$ ft, $M_{cx} = \phi M_{nx} = 642$ kip-ft	at $L_b = 14.0$ ft, $M_{cx} = M_{nx}/\Omega = 428$ kip-ft	Manual Table 3-10
$M_{cy} = \phi M_{ny} = 311$ kip-ft	$M_{cy} = M_{ny}/\Omega = 207$ kip-ft	Manual Table 3-2
$\frac{P_u}{\phi_c P_n} = \frac{400\text{ kips}}{1{,}130\text{ kips}} = 0.354$	$\frac{P_a}{P_n/\Omega_c} = \frac{267\text{ kips}}{751\text{ kips}} = 0.356$	
Since $\frac{P_u}{\phi_c P_n} > 0.2$, use Eqn. H1.1a	Since $\frac{P_a}{P_n/\Omega_c} > 0.2$, use Eqn. H1.1a	
$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$	$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$	Eq. H1.1a
$\frac{400\text{ kips}}{1130\text{ kips}} + \frac{8}{9}\left(\frac{250\text{ kip-ft}}{642\text{ kip-ft}} + \frac{80.0\text{ kip-ft}}{311\text{ kip-ft}}\right)$ $= 0.354 + \frac{8}{9}(0.389 + 0.257) = 0.929 < 1.0$ **o.k.**	$\frac{267\text{ kips}}{751\text{ kips}} + \frac{8}{9}\left(\frac{167\text{ kip-ft}}{428\text{ kip-ft}} + \frac{53.3\text{ kip-ft}}{207\text{ kip-ft}}\right)$ $= 0.356 + \frac{8}{9}(0.390 + 0.257) = 0.931 < 1.0$ **o.k.**	

ASDIP Steel 3.2.5 | **STEEL COLUMN DESIGN** | *www.asdipsoft.com*

GEOMETRY

Column Designation	W14X99		
Steel Yield Strength Fy	50.0	ksi	
Modulus of Elasticity Es ...	29000	ksi	
Member Length L	14.00	ft	
Effective Length Kx-factor	1.00		
Effective Length Ky-factor	1.00		
Unbraced Length Lb	14.00	ft	OK

PROPERTIES

Area ..	29.1	in²	Sx ...	157.0	in³
Depth	14.2	in	Zx ...	173.0	in³
bf	14.6	in	rx	6.17	in
tw	0.49	in	Iy	402.0	in⁴
tf	0.78	in	Sy ...	55.2	in³
k des .	1.38	in	Zy ...	83.6	in³
Ix	1110.0	in⁴	ry	3.71	in

SERVICE LOADS (ASD)

Loads from a General 2nd-Order Analysis

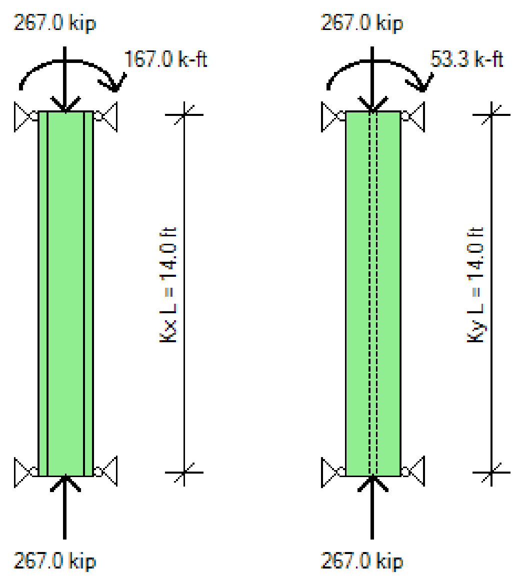
Axial Load P	267.0	kip
Max. Moment x-x	167.0	k-ft
Max. Moment y-y	53.3	k-ft

LOCAL BUCKLING

Flanges in Flexure	Non-compact
Flanges in Compression	Non-slender
Web in Flexure	Compact
Web in Compression	Non-slender

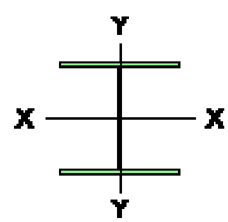
COMPRESSION

Slenderness Ratio Kx L / rx	27.2		
Slenderness Ratio Ky L / ry	45.3		
Max. Slenderness Ratio	45.3		OK
Limit States	Nominal Pn		
Flexural Buckling	1252.4	kip	✓
Torsional Buckling	1277.1	kip	
Flexural-Torsional Buckling	N.A.	kip	
Nominal Strength Pn	1252.4	kip	
Safety Factor Ω	1.67		
Allowable Strength Pn/Ω	750.0	kip	
P / Pn/Ω Design Ratio	0.36		OK



BENDING ABOUT Y-Y

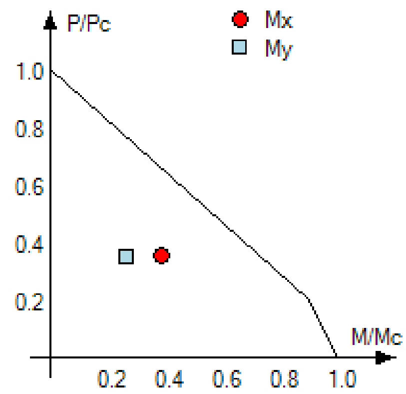
Limit States	Nominal Mn		
Yielding	348.3	k-ft	
Lateral-Torsional Buckling	N.A.	k-ft	
Flange Local Buckling	345.7	k-ft	✓
Web Local Buckling	N.A.	k-ft	
Nominal Strength Mn	345.7	k-ft	
Safety Factor Ω	1.67		
Allowable Strength Mn/Ω ...	207.0	k-ft	
M / Mn/Ω Design Ratio	0.26		OK



STEEL COLUMN DESIGN

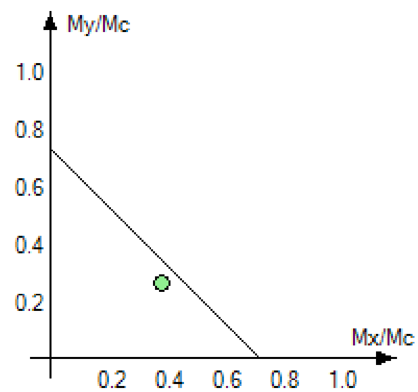
BENDING ABOUT X-X

Moment at 1/4 point of Lb ...	N.A.	k-ft	
Moment at 1/2 point of Lb ...	N.A.	k-ft	
Moment at 3/4 point of Lb ...	N.A.	k-ft	
L. T. Buckling Cb-factor	1.75		
Limit States	Nominal Mn		
Yielding	N.A.	k-ft	
Lateral-Torsional Buckling	720.8	k-ft	
Flange Local Buckling	717.2	k-ft	✓
Web Local Buckling	N.A.	k-ft	
Nominal Strength Mn	717.2	k-ft	
Safety Factor Ω	1.67		
Allowable Strength Mn/Ω ...	429.5	k-ft	
M / Mn/Ω Design Ratio	0.39		OK



COMBINED FORCES

AISC Equation {H1-1a)	0.93	OK
AISC Equation {H1-1b)	N.A.	



ASDIP Steel 3.2.5 | **STEEL COLUMN DESIGN** | www.asdipsoft.com

GEOMETRY

Column Designation	W14X99	
Steel Yield Strength Fy	50.0	ksi
Modulus of Elasticity Es	29000	ksi
Member Length L	14.00	ft
Effective Length Kx-factor ..	1.00	
Effective Length Ky-factor ..	1.00	
Unbraced Length Lb	14.00	ft OK

PROPERTIES

Area ..	29.1	in^2	Sx ...	157.0	in^3
Depth	14.2	in	Zx ...	173.0	in^3
bf	14.6	in	rx	6.17	in
tw	0.49	in	Iy	402.0	in^4
tf	0.78	in	Sy ...	55.2	in^3
k des .	1.38	in	Zy ...	83.6	in^3
Ix	1110.0	in^4	ry	3.71	in

SERVICE LOADS (ASD)

Loads from a General 2nd-Order Analysis

Axial Load P	267.0	kip
Max. Moment x-x	167.0	k-ft
Max. Moment y-y	53.3	k-ft

LOCAL BUCKLING

Flanges in Flexure	Non-compact
Flanges in Compression	Non-slender
Web in Flexure	Compact
Web in Compression	Non-slender

DESIGN FOR COMPRESSION

Slender unstiffened Qs = 1.00 , Slender stiffened Qa = 1.00 , Q = Qs * Qa = 1.00 — AISC E7

Slenderness ratio $Kx\,L / rx =$ 1.00 * 14.00 / 6.17 = 27.2

Slenderness ratio $Ky\,L / ry =$ 1.00 * 14.00 / 3.71 = 45.3

Maximum slenderness ratio = Max (27.2, 45.3) = 45.3 — AISC E3

Elastic buckling $Fe = \dfrac{\pi^2 * E}{(K\,L / r)^2} = \dfrac{3.14^2 * 29000}{45.3^2} = 139.6$ ksi — AISC Eq. E3-4

$Fe >= 0.44 * Q * Fy$ (139.6 >= 0.44 * 1.00 * 50.0 = 22.0)

Flexural buckling stress $Fcr = Q * Fy * (0.658)^{Q * Fy / Fe}$ — AISC Eq. E3-2

$Fcr = 1.00 * 50.0 * (0.658)^{1.00 * 50.0 / 139.6} = 43.0$ ksi

$$Fe = \left[\frac{\pi^2 * E * Cw}{(Ky\,L)^2} + G * J \right] \frac{1}{Ix + Iy}$$ — AISC Eq. E4-4

$$= \left[\frac{\pi^2 * 29000 * 18000.0}{(1.00 * 14.0 * 12)^2} + 11200 * 5.4 \right] \frac{1}{1110.0 + 402.0} = 160.5 \text{ ksi}$$

$Fe >= 0.44 * Q * Fy$ (160.5 >= 0.44 * 1.00 * 50.0 = 22.0)

Torsional buckling $Fcr = Q * Fy * (0.658)^{Q * Fy / Fe}$ — AISC Eq. E3-2

$Fcr = 1.00 * 50.0 * (0.658)^{1.00 * 50.0 / 160.5} = 43.9$ ksi

Compressive strength $Pn = Fcr * Ag =$ 43.0 * 29.1 = 1252.4 kip — AISC Eq. E3-1

Controlling limit state: Flexural Buckling

Compressive design ratio = $\dfrac{P}{Pn / \Omega} = \dfrac{267.0}{1252.4 / 1.67} = 0.36 < 1.0$ OK — AISC E1

DESIGN FOR FLEXURE

Cb = 1.75 (User-defined)

- Lateral-Torsional Buckling

$Lp = 1.76 \cdot ry \cdot \sqrt{\frac{E}{Fy}} = 1.76 \cdot 3.7 \cdot \sqrt{\frac{29000}{50}}$ Lp = 157.3 in — AISC Eq. F2-5

$rts = \sqrt{\frac{\sqrt{Iy \cdot Cw}}{Sx}} = \sqrt{\frac{\sqrt{402 \cdot 18000}}{157}}$ rts = 4.1 in — AISC Eq. F2-7

ho = d - tf = 14.2 - 0.8 = 13.4 in

c = 1.0 (doubly symmetric I-shape) — AISC Eq. F2-8a

$Lr = \frac{1.95 \cdot rts \cdot E}{(0.7 \cdot Fy)} \cdot \sqrt{\frac{J \cdot c}{(Sx \cdot ho)} + \sqrt{\left(\frac{J \cdot c}{(Sx \cdot ho)}\right)^2 + 6.76 \cdot \left(\frac{0.7 \cdot Fy}{E}\right)^2}}$ — AISC Eq. F2-6

$\frac{1.95 \cdot 4.1 \cdot 29000}{0.7 \cdot 50} \cdot \sqrt{\frac{5.4 \cdot 1}{157 \cdot 13.4} + \sqrt{\frac{5.4 \cdot 1}{157 \cdot 13.4}^2 + 6.76 \cdot \frac{0.7 \cdot 50}{29000}^2}}$ Lr = 543.0 in

$Fcr = \frac{Cb * \pi^2 * E}{(Lb / rts)^2} \sqrt{1 + \frac{0.078 \cdot J \cdot c}{(Sx \cdot ho)} \cdot \left(\frac{Lb}{rts}\right)^2}$ — AISC Eq. F2-4

$= \frac{1.75 * \pi^2 * 29000.0}{(14.0 * 12 / 4.1)^2} \sqrt{1 + \frac{0.078 \cdot 5.4 \cdot 1}{157 \cdot 13.4} \cdot \frac{14}{4.1}^2}$ Fcr = 350.3 ksi

Nominal strength *Mnx = Cb * [Mpx - (Mpx - 0.7 * Fy * Sx) * (Lb - Lp) / (Lr - Lp)]* — AISC Eq. F2-2

Mnx = 1.75 * [8650.0 - (8650.0 - 0.7 * 50.0 * 157.0)(14.0 * 12 - 157.3) / (543.0 - 157.3)] / 12 = 720.8 k-ft

- Compression Flange Local Buckling

*λ = bf / (2 * tf) =* 14.6 / (2 * 0.8) = 9.4

$\lambda_{pf} = 0.38 \cdot \sqrt{\frac{E}{Fy}} = 0.38 * \sqrt{\frac{29000}{50}} = 9.2$

$\lambda_{rf} = 1.0 * \sqrt{\frac{E}{Fy}} = 1.0 * \sqrt{\frac{29000}{50}} = 24.1$

*Mnx = (Mpx - (Mpx - 0.7 * Fy * Sx) * (λ - λpf) / (λrf - λpf)) / 12* — AISC Eq F3-1

= (8650.0 - (8650.0 - 0.7 * 50.0 * 157.0) * (9.4 - 9.2) / (24.1 - 9.2)) / 12 = 717.2 k-ft

X - Controlling limit state: Flange Local Buckling

X - Flexural design ratio = $\frac{Mx}{Mnx / \Omega} = \frac{167.0}{717.2 / 1.67}$ = 0.39 < 1.0 OK — AISC F1

- Yielding

*Mpy = Min (Fy * Zy, 1.6 * Fy * Sy) =*Min (50.0 * 83.6, 1.6 * 50.0 * 55.2) = 4180.0 k-in

Nominal strength *Mny = Mpy =* 4180.0 / 12 = 348.3 k-ft

- Flange Local Buckling — AISC Eq. F6-1

*Mny = (Mpy - (Mpy - 0.7 * Fy * Sy) * (λ - λpf) / (λrf - λpf)) / 12* — AISC Eq F6-2

= (4180.0 - (4180.0 - 0.7 * 50.0 * 55.2) * (9.4 - 9.2) / (24.1 - 9.2)) / 12 = 345.7 k-ft

Y - Controlling limit state: Flange Local Buckling

Y - Flexural design ratio = $\frac{My}{Mny / \Omega} = \frac{53.3}{345.7 / 1.67}$ = 0.26 < 1.0 OK — AISC F1

DESIGN FOR COMBINED FORCES

Allowable axial strength $Pc = \dfrac{Pn}{\Omega} = \dfrac{1252.4}{1.67} = 750.0$ kip — AISC E1

Allowable flexural strength $Mcx = \dfrac{Mnx}{\Omega} = \dfrac{717.2}{1.67} = 429.5$ kip — AISC F1

Allowable flexural strength $Mcy = \dfrac{Mny}{\Omega} = \dfrac{345.7}{1.67} = 207.0$ kip — AISC F1

Combined forces ratio $= \dfrac{P}{Pc} + 8/9\left[\dfrac{Mx}{Mcx} + \dfrac{My}{Mcy}\right]$ — AISC Eq. H1-1a

$$= \frac{267.0}{750.0} + 8/9\left[\frac{167.0}{429.5} + \frac{53.3}{207.0}\right] = 0.93 < 1.0$$ OK