ASDIP Concrete

STRUCTURAL ENGINEERING SOFTWARE

Non-sway Column Example

PCA Notes on ACI 318

Example 11.1—Slenderness Effects for Columns in a Nonsway Frame

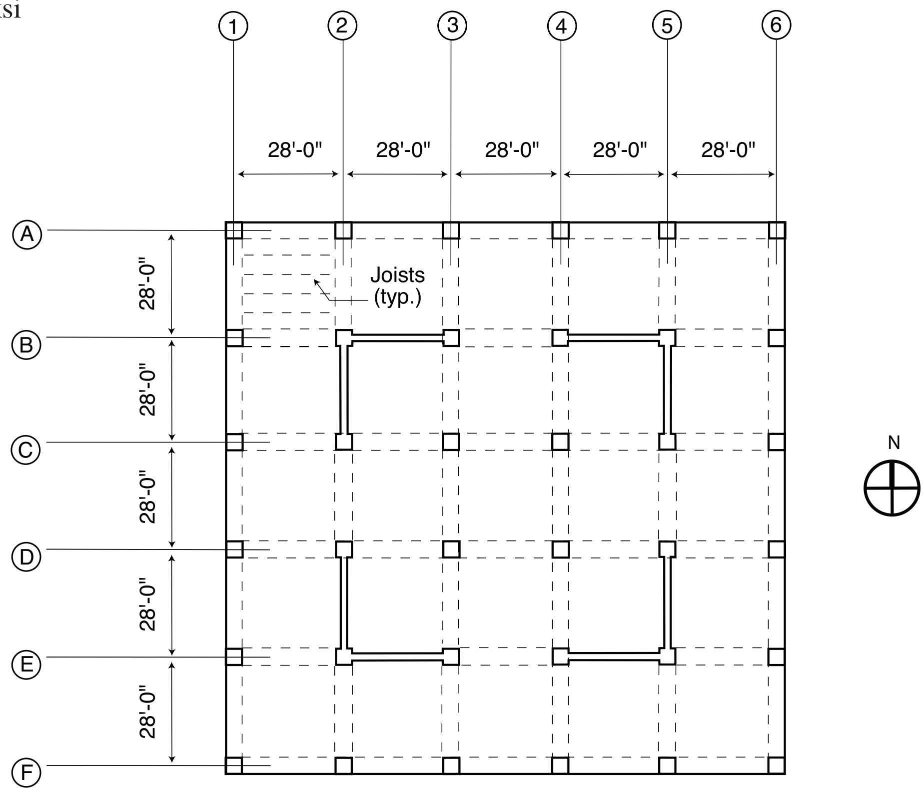
Design columns A3 and C3 in the first story of the 10-story office building shown below. The clear height of the first story is 21 ft-4 in., and is 11 ft-4 in. for all of the other stories. Assume that the lateral load effects on the building are caused by wind, and that the dead loads are the only sustained loads. Other pertinent design data for the building are as follows:

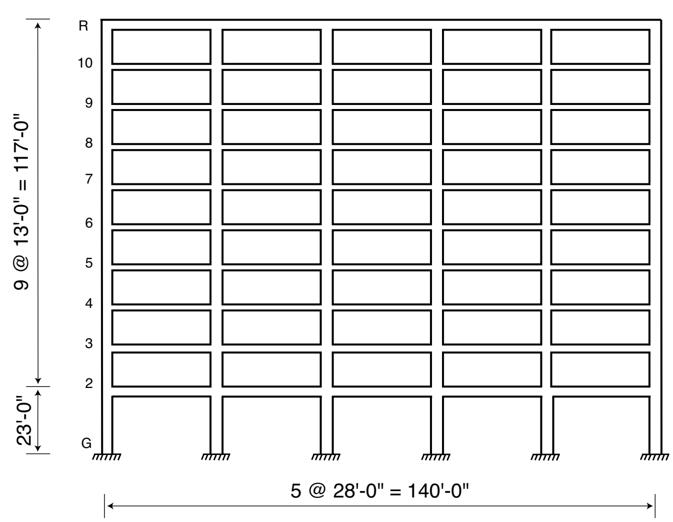
Material properties:

Concrete:
Floors: $f'_c = 4000$ psi, $w_c = 150$ pcf
Columns and walls: $f'_c = 6000$ psi, $w_c = 150$ pcf
Reinforcement: $f_y = 60$ ksi

Beams: 24 × 20 in.
Exterior columns: 20 × 20 in.
Interior columns: 24 × 24 in.
Shearwalls: 12 in.

Weight of floor joists = 86 psf
Superimposed dead load = 32 psf
Roof live load = 30 psf
Floor live load = 50 psf
Wind loads computed according to ASCE 7.





1. Factored axial loads and bending moments for columns A3 and C3 in the first story

Column A3

Eq.	No.	Load Case	Axial Load (kips)	Bending Moment (ft-kips) Top	Bottom
		Dead (D)	718.0	79.0	40.0
		Live (L)*	80.0	30.3	15.3
		Roof live load (L_r)	12.0	0.0	0.0
		Wind (W)	±8.0	±1.1	±4.3
Eq.	No.	Load Combination			
9-1	1	1.4D	1,005.2	110.6	56.0
9-2	2	1.2D + 1.6L + 0.5L_r	995.6	143.3	72.5
9-3	3	1.2D + 0.5L + 1.6L_r	920.8	110.0	55.7
	4	1.2D + 1.6L_r + 0.8W	887.2	95.7	51.4
	5	1.2D + 1.6L_r - 0.8W	874.4	93.9	44.6
9-4	6	1.2D + 0.5L + 0.5L_r + 1.6W	920.4	111.7	62.5
	7	1.2D + 0.5L + 0.5L_r - 1.6W	894.8	108.2	48.8
9-6	8	0.9D + 1.6W	659.0	72.9	42.9
	9	0.9D - 1.6W	633.4	69.3	29.1

*includes live load reduction per ASCE 7

Column C3

Eq.	No.	Load Case	Axial Load (kips)	Bending Moment (ft-kips) Top	Bottom
		Dead (D)	1,269.0	1.0	0.7
		Live (L)*	147.0	32.4	16.3
		Roof live load (L_r)	24.0	0.0	0.0
		Wind (W)	±3.0	±2.5	±7.7
Eq.	No.	Load Combination			
9-1	1	1.4D	1,776.6	1.4	1.0
9-2	2	1.2D + 1.6L + 0.5L_r	1,770.0	53.0	26.9
9-3	3	1.2D + 0.5L + 1.6L_r	1,634.7	17.4	9.0
	4	1.2D + 1.6L_r + 0.8W	1,563.6	3.2	7.0
	5	1.2D + 1.6L_r - 0.8W	1,558.8	-0.8	-5.3
9-4	6	1.2D + 0.5L + 0.5L_r + 1.6W	1,613.1	21.4	21.3
	7	1.2D + 0.5L + 0.5L_r - 1.6W	1,603.5	13.4	-3.3
9-6	8	0.9D + 1.6W	1,146.9	4.9	13.0
	9	0.9D - 1.6W	1,137.3	-3.1	-11.7

*includes live load reduction per ASCE 7

Note that Columns A3 and C3 are bent in double curvature with the exception of Load Case 7 for Column C3.

2. Determine if the frame at the first story is nonsway or sway

The results from an elastic first-order analysis using the section properties prescribed in 10.10.4.1 are as follows:

ΣP_u = total vertical load in the first story corresponding to the lateral loading case for which ΣP_u is greatest

The total building loads are: D = 37,371 kips, L = 3609 kips, and L_r = 605 kips.
The maximum ΣP_u is determined from Eq. (9-4):

$$\Sigma P_u = (1.2 \times 37{,}371) + (0.5 \times 3609) + (0.5 \times 605) + 0 = 46{,}952 \text{ kips}$$

V_{us} = factored story shear in the first story corresponding to the wind loads
= 1.6 × 324.3 = 518.9 kips — *Eq. (9-4), (9-6)*

Δ_o = first-order relative lateral deflection between the top and bottom of the first story due to V_{us}
= 1.6 × (0.03-0) = 0.05 in.

$$\text{Stability index } Q = \frac{\Sigma P_u \Delta_o}{V_{us}\ell_c} = \frac{46{,}952 \times 0.05}{518.9 \times \left[(23\times 12)-(20/2)\right]} = 0.02 < 0.05$$ — *Eq. (10-10)*

Since Q < 0.05, the frame at the first story level is considered nonsway. — *10.10.5.2*

3. Design of column C3

Determine if slenderness effects must be considered.

Using an effective length factor k = 1.0, — *10.10.6.3*

$$\frac{k\ell_u}{r} = \frac{1.0 \times 21.33 \times 12}{0.3 \times 24} = 35.6$$

The following table contains the slenderness limit for each load case:

Eq.	No.	Axial loads (kips)	Bending Moment (ft-kips)		Curvature	M_1 (ft-kips)	M_2 (ft-kips)	M_1/M_2	Slenderness* limit
		P_u	M_{top}	M_{bot}					
9-1	1	1776.6	1.4	1.0	Double	1.0	1.4	–0.70	40.00
9-2	2	1770.0	53.0	26.9	Double	26.9	53.0	–0.51	40.00
9-3	3	1634.7	17.4	9.0	Double	9.0	17.4	–0.52	40.00
	4	1564.2	3.7	8.5	Double	3.7	8.5	–0.43	39.20
	5	1558.2	–1.3	–6.9	Double	1.3	6.9	–0.19	36.27
9-4	6	1613.1	21.4	21.3	Double	21.3	21.4	–1.00	40.00
	7	1603.5	13.4	-3.3	Single	3.3	13.4	+0.25	31.02
9-6	8	1146.9	4.9	13.0	Double	4.9	13.0	–0.38	38.54
	9	1137.3	–3.1	–11.7	Double	3.1	11.7	–0.27	37.18

* $34 - 12\left(\frac{M_1}{M_2}\right) \le 40$

The least value of $34 - 12\left(\frac{M_1}{M_2}\right)$ is obtained from load combination no. 7:

$$34 - 12\left[\frac{M_1}{M_2}\right] = 34 - 12\left[\frac{3.3}{13.4}\right] = 31.02 < 40$$

Slenderness effects need to be considered for column C3 since $k\ell_u/r > 34 - 12\,(M_1/M_2)$. 10.12.2

The following calculations illustrate the magnified moment calculations for load combination no. 7:

$$M_c = \delta_{ns} M_2 \qquad \text{Eq. (10-11)}$$

where

$$\delta_{ns} = \frac{C_m}{1 - \dfrac{P_u}{0.75P_c}} \geq 1 \qquad \text{Eq. (10-12)}$$

$$C_m = 0.6 + 0.4\left(\frac{M_1}{M_2}\right) \geq 0.40 \qquad \text{Eq. (10-16)}$$

$$= 0.6 + 0.4\left(\frac{3.3}{13.4}\right) = 0.70$$

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \qquad \text{Eq. (10-13)}$$

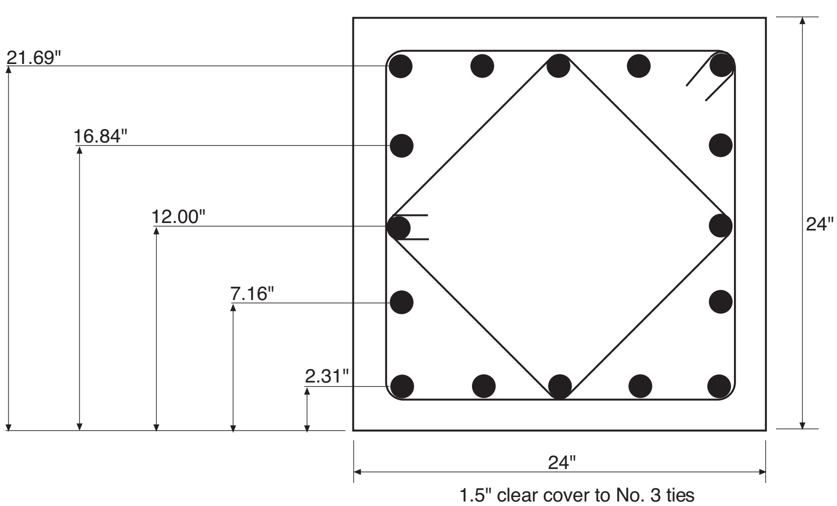
$$E1 = \frac{0.2E_cI_g + E_sI_e}{1+\beta_{dns}} \qquad \text{Eq. (10-14)}$$

$$E_c = 57{,}000\frac{\sqrt{6000}}{1000} = 4415 \text{ ksi}$$

$$I_g = \frac{24^4}{12} = 27{,}648 \text{ in.}^4$$

$$E_s = 29{,}000 \text{ ksi}$$

Assuming 16-No. 7 bars with 1.5 cover to No. 3 ties as shown in the figure.



$$I_{se} = 2\left[(5\times 0.6)(21.69-12)^2 + (2\times 0.6)(16.84-12)^2\right]$$

$$= 619.6 \text{ in.}^4$$

Since the dead load is the only sustained load,

$$\beta_{dns} = \frac{1.2P_D}{1.2P_D + 0.5P_L + 0.5P_{Lr} - 1.6W} \leq 1$$ 10.10.6.2

$$= \frac{1.2\times 1269}{(1.2\times 1269)+(0.5\times 147)+(0.5\times 24)-(1.6\times 3)}$$

$$= 0.95$$

$$EI = \frac{(0.2\times 4415\times 27{,}648)+(29{,}000\times 619.6)}{1+0.95} = 21.73\times 10^6 \text{ kip-in.}^2$$

$$P_c = \frac{\pi^2\times 21.73\times 10^6}{(1\times 21.33\times 12)^2} = 3274 \text{ kips}$$

$$\delta_{ns} = \frac{0.7}{1-\dfrac{1603.5}{0.75\times 3274}} = 2.02$$ (see "Closing Remarks" at the end of the Example)

Check miminum moment requirement:

$M_{2,\,min} = P_n(0.6 + 0.03h)$

$= 1603.5[0.6+(0.03 \times 24)]/12$

$= 176.4$ ft-kip $> M_2$

$M_2 = 2.02 \times 176.4 = 356.3$ ft-kip

The following table contains results from a strain compatibility analysis, where compressive strains are taken as positive (see Part 6 and 7).

Therefore, since $\phi M_n > M_u$ for all $\phi P_n = P_u$, use a 24 $\times$ 24 in. column with 16-No. 7 bars (ρ_g =1.7%).

No.	P_u (kips)	M_u (ft-kips)	c (in.)	ε_t	ϕ	ϕP_n (kips)	ϕM_n (ft-kips)
1	1776.6	1.4	25.92	0.00049	0.65	1776.6	367.2
2	1770.0	53.0	25.83	0.00048	0.65	1770.0	371.0
3	1634.7	17.4	23.86	0.00027	0.65	1634.7	447.0
4	1563.6	7.0	22.85	0.00015	0.65	1563.6	480.9
5	1558.8	5.3	22.78	0.00014	0.65	1558.8	483.2
6	1613.1	21.4	23.55	0.00024	0.65	1613.1	457.8
7	1603.5	356.3	23.41	0.00022	0.65	1603.5	462.5
8	1146.9	13.0	17.25	-0.00077	0.65	1146.9	609.9
9	1137.3	11.7	17.13	-0.00080	0.65	1137.3	611.7

Design for P_u and M_c can be performed manually, by creating an interaction diagram as shown in example 6.4. For this example, Figure 11-14 shows the design srength interaction diagram for Column C3 obtained from the computer program pcaColumn. The figure also shows the axial load and moments for load combination 7.

4. Design of column A3

   a. Determine if slenderness effects must be considered.

   Determine k from the alignment chart of Fig. 11-9 or from Fig. R10.10.1.1:

$$I_{col} = 0.7\left(\frac{20^4}{12}\right) = 9{,}333 \text{ in.}^4$$ 10.10.4.1

$$E_c = 57{,}000\frac{\sqrt{6000}}{1000} = 4{,}415 \text{ ksi}$$ 8.5.1

For the column below level 2:

$$\left(\frac{E_c I}{\ell_c}\right) = \frac{4{,}415\times 9{,}333}{[(23\times 12)-(20/2)]} = 155 \times 10^3 \text{ in.-kips}$$

For the column above level 2:

$$\left(\frac{E_c I}{\ell_c}\right) = \frac{4{,}415\times 9{,}333}{13\times 12} = 264 \times 10^3 \text{ in.-kips}$$

10.10.4.1

$$I_{beam} = 0.35\left(\frac{24\times 20^3}{12}\right) = 5{,}600 \text{ in.}^4$$

$$\frac{EI}{\ell} = \frac{57\sqrt{4{,}000}\times 5{,}600}{28\times 12} = 60 \times 10^3 \text{ in.-kips}$$

$$\psi_A = \frac{\Sigma E_c I/\ell_c}{\Sigma E_c I/\ell} = \frac{155+264}{60} = 60 \times 10^3 \text{ in.-kips}$$

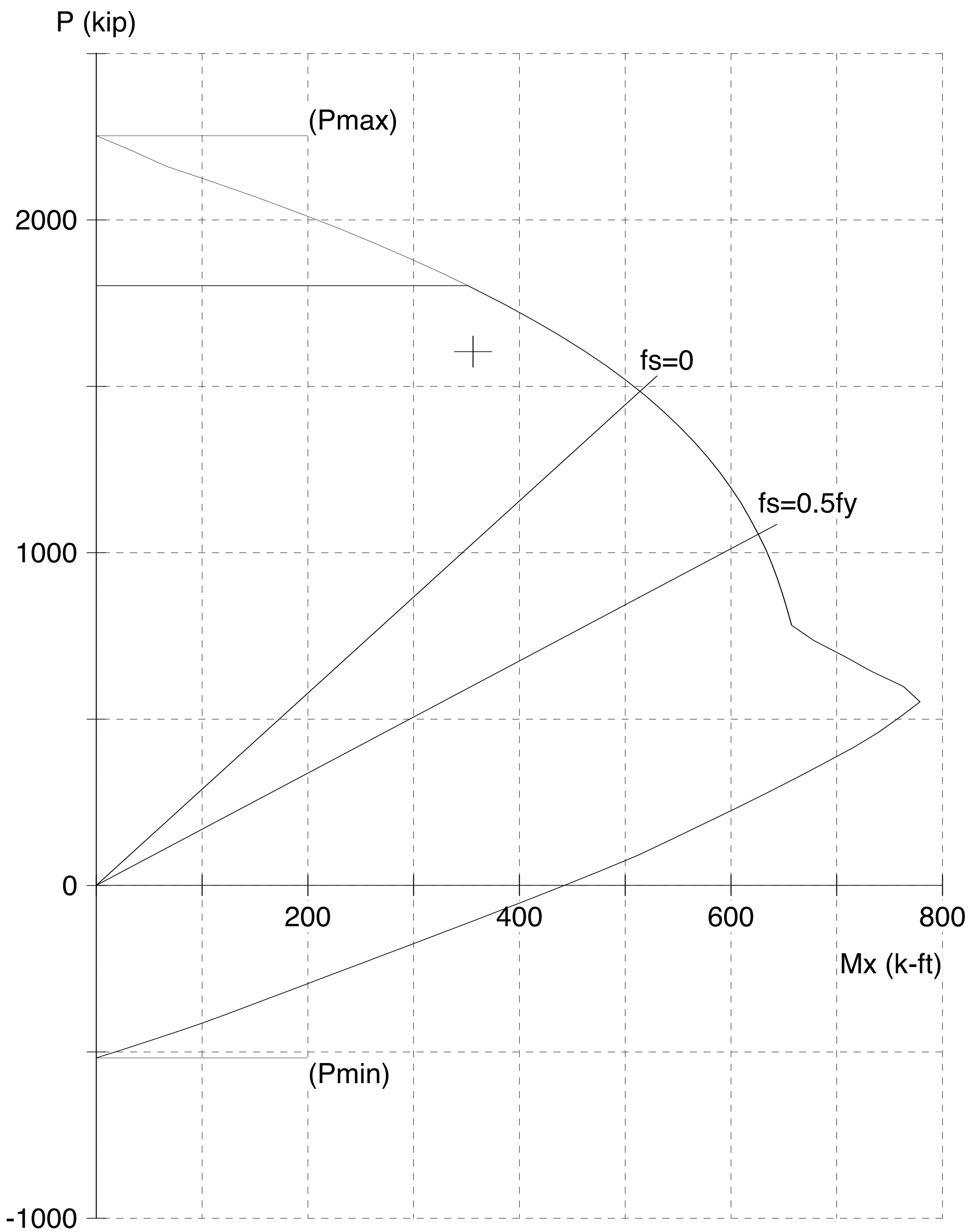


Figure 11-14 Interaction Diagram for Column C3

ASDIP Concrete 2.0.0 **CONCRETE COLUMN DESIGN** *www.asdipsoft.com*

GEOMETRY

Width .... 24.0 in , Height ... 24.0 in
Use 16-#7 Longitudinal Bars , ρ = 0.017 OK
Use Ties #3 Transverse Reinf. , d' = 2.3 in
Concrete Clear Cover .... 1.5 in

SLENDERNESS

Clear Member Length Lu .... 21.33 ft
Effective Length Kx-factor 1.00
Effective Length Ky-factor 1.00
Lateral Stability .... *Non-sway Column*
Slenderness Ratio = 37 > Limit = 31, Slender

UNFACTORED LOADS (Elastic First-Order Analysis)

	Dead	Live	RLive	Snow	Wind	Seismic	
Axial Force P	1269.0	147.0	24.0	0.0	-3.0	0.0	kip
Mx at Bottom	0.7	16.3	0.0	0.0	-7.7	0.0	k-ft
Mx at Top	-1.0	-32.4	0.0	0.0	2.5	0.0	k-ft

AMPLIFIED FACTORED LOADS (Non-sway Column)

	Load Combination	Pu (kip)	M1x (k-ft)	M2x (k-ft)	M1x/ M2x	Cmx	δnsx *	Mcx (k-ft)	
1	1.4D	1776.6	-1.0	195.4	-0.70	0.32	6.40	1250.7	NG
2	1.2D+1.6L+0.5Lr	1770.0	-26.9	194.7	-0.51	0.40	7.94	1545.9	NG
3	1.2D+1.6L+0.5S	1758.0	-26.9	193.4	-0.51	0.40	7.94	1535.4	NG
4	1.2D+0.5L+1.6Lr	1634.7	-9.0	179.8	-0.52	0.39	7.87	1414.6	NG
5	1.2D+0.5L+1.6S	1596.3	-9.0	175.6	-0.52	0.39	7.87	1381.3	NG
6	1.2D+1.6Lr+0.8W	1558.8	-0.8	171.5	-0.15	0.54	10.80	1851.3	NG
7	1.2D+1.6S+0.8W	1520.4	-0.8	167.2	-0.15	0.54	10.80	1805.7	NG
8	1.2D+0.5L+0.5Lr+1.6W	1603.5	3.3	176.4	0.25	0.70	13.99	2467.3	NG
9	1.2D+0.5L+0.5S+1.6W	1591.5	3.3	175.1	0.25	0.70	13.99	2448.8	NG
10	1.2D+0.5L+0.2S+1.0E	1596.3	-9.0	175.6	-0.52	0.39	7.87	1381.3	NG
11	0.9D+1.6W	1137.3	-3.1	125.1	-0.27	0.49	2.84	355.8	NG
12	0.9D+1.0E	1142.1	-0.6	125.6	-0.70	0.32	1.86	233.8	NG

* Per ACI 10.10.2.1, δns cannot be greater than 1.4

DESIGN CODES

Concrete Design .... ACI 318-11
Load Combinations .... ASCE 7-05

MATERIALS

Concrete Strength f'c .... 6.0 ksi
Rebar Steel Strength fy ...... 60.0 ksi
Compression Strain Limit 0.003

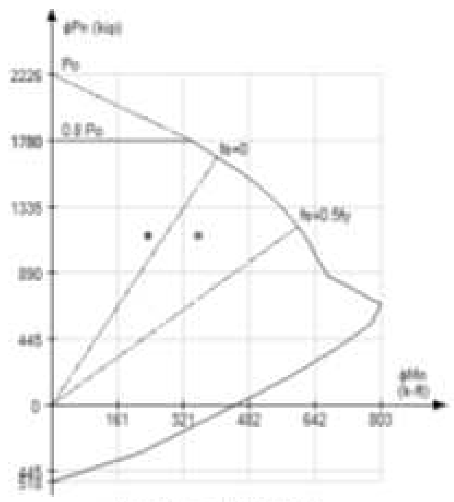
CONCRETE COLUMN DESIGN

INTERACTION DIAGRAM

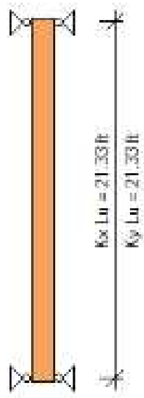
Condition	k = c/d	Steel Fs (kip)	Steel Ms (k-ft)	Conc Fc (kip)	Conc Mc (k-ft)	Pn (kip)	Mn (k-ft)
Pure Compression	Inf.	527.7	0.0	2896.1	0.0	2225.5	0.0
Max. Usable Axial	1.07	314.0	137.6	2425.1	387.7	2739.1	525.4
Zero Steel Stress	1.00	289.4	150.3	2284.2	471.2	2573.6	621.4
Steel Stress = 0.5 fy	0.74	144.8	228.1	1698.4	696.0	1843.3	924.2
Balanced Condition	0.59	-1.9	307.4	1351.9	716.8	1350.0	1024.2
Steel Strain = 0.005	0.38	-127.1	311.0	856.4	601.6	729.3	912.6
Pure Bending	0.15	0.0	201.4	0.0	298.8	0.0	500.1
Pure Tension	0.00	518.4	0.0	0.0	0.0	518.4	0.0

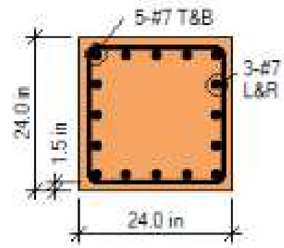
COLUMN STRENGTH

Comb	Pu (kip)	Mux (k-ft)	k = c/d	Steel Fs (kip)	Steel Ms (k-ft)	Conc Fc (kip)	Conc Mc (k-ft)	ϕ Factor	ϕPn (kip)	ϕMn (k-ft)	
1	1776.6	1250.7	1.07	313.1	138.1	2420.1	390.7	0.65	1776.6	343.8	NG
2	1770.0	1545.9	1.06	311.6	139.0	2411.4	395.9	0.65	1770.0	347.7	NG
3	1758.0	1535.4	1.05	308.8	140.4	2395.8	405.2	0.65	1758.0	354.7	NG
4	1634.7	1414.6	0.98	280.1	155.2	2234.8	499.3	0.65	1634.7	425.5	NG
5	1596.3	1381.3	0.96	270.4	160.5	2185.4	525.8	0.65	1596.3	446.1	NG
6	1558.8	1851.3	0.94	260.6	165.7	2137.5	549.8	0.65	1558.8	465.1	NG
7	1520.4	1805.7	0.91	250.2	171.3	2088.9	572.7	0.65	1520.4	483.6	NG
8	1603.5	2467.3	0.96	272.3	159.5	2194.7	521.0	0.65	1603.5	442.3	NG
9	1591.5	2448.8	0.95	269.2	161.1	2179.3	529.0	0.65	1591.5	448.6	NG
10	1596.3	1381.3	0.96	270.4	160.5	2185.4	525.8	0.65	1596.3	446.1	NG
11	1137.3	355.8	0.71	120.4	241.3	1629.2	706.9	0.65	1137.3	616.3	OK
12	1142.1	233.8	0.72	122.4	240.2	1634.6	706.1	0.65	1142.1	615.1	OK



INTERACTION DIAGRAM





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