Unbraced Column Verification Example

AISC Design Examples
AISC 13^{th} Edition

Example H.4 W-Shape Subject to Combined Axial Compression and Flexure

Given: Select an ASTM A992 W-shape with a 10 in. nominal depth to carry nominal axial compression forces of 5 kips from dead load and 15 kips from live load. The unbraced length is 14 ft and the ends are pinned. The member also has the following nominal required moment strengths, not including second-order effects:

M_{xD} = 15 kip-ft M_{xL} = 45 kip-ft
M_{yD} = 2 kip-ft M_{yL} = 6 kip-ft

The member is not subject to sidesway.

Solution:

Material Properties:
ASTM A992 F_y = 50 ksi F_u = 65 ksi

Calculate the required strength, not considering second-order effects

LRFD	ASD
P_u = 1.2(5.00 kips) + 1.6(15.0 kips) = 30.0 kips M_{ux} = 1.2(15.0 kip-ft) + 1.6(45.0 kip-ft) = 90.0 kip-ft M_{uy} = 1.2(2.00 kip-ft) + 1.6(6.00 kip-ft) = 12.0 kip-ft	P_a = 5.00 kips + 15.0 kips = 20.0 kips M_{ax} = 15.0 kip-ft + 45.0 kip-ft = 60.0 kip-ft M_{ay} = 2.00 kip-ft + 6.00 kip-ft = 8.00 kip-ft

Try a W10×33

Geometric Properties:

W10×33 A = 9.71 in.^{2} S_x= 35.0 in.^{3} Z_x = 38.8 in.^{3} I_x = 171 in.^{4} Manual
S_y= 9.20 in.^{3} Z_y = 14.0 in.^{3} I_y = 36.6 in.^{4} Table 1-1
L_p = 6.85 ft L_r = 12.8 ft Table 3-1

Calculate the available axial strength

For a pinned-pinned condition, K = 1.0. Commentary Table C-C2.2

Since KL_x = KL_y = 14.0 ft and r_x > r_y, the y-y axis will govern.

Manual Table 4-1

LRFD	ASD
$P_c = \phi_c P_n$ = 253 kips	$P_c = P_n/\Omega_c$ = 168 kips

Calculate the required flexural strengths including second order amplification

Use “Amplified First-Order Elastic Analysis” procedure from Section C2.1b. Since the member is not subject to sidesway, only P-δ amplifiers need to be added.

$$B_1 = \frac{C_m}{1-\alpha P_r / P_{e1}}$$ Eqn. C2-2

C_m = 1.0

X-X axis flexural magnifier

$$P_{e1} = \frac{\pi^2 EI_x}{(K_1 L_x)^2} = \frac{\pi^2 (29{,}000\text{ ksi})(171\text{ in.}^4)}{((1.0)(14.0\text{ ft})(12\text{ in./ft}))^2} = 1730\text{ kips}$$ Eqn. C2-5

LRFD	ASD
α = 1.0 $B_1 = \dfrac{1.0}{1-1.0(30.0\text{ kips}/1730\text{ kips})} = 1.02$ M_{ux} = 1.02(90.0 kip-ft) = 91.8 kip-ft	α = 1.6 $B_1 = \dfrac{1.0}{1-1.6(20.0\text{ kips}/1730\text{ kips})} = 1.02$ M_{ax} = 1.02(60.0 kip-ft) = 61.2 kip-ft

Eqn. C2-2

Y-Y axis flexural magnifier

$$P_{e1} = \frac{\pi^2 EI_y}{(K_1 L_y)^2} = \frac{\pi^2 (29{,}000\text{ ksi})(36.6\text{ in.}^4)}{((1.0)(14.0\text{ ft})(12\text{ in./ft}))^2} = 371\text{ kips}$$ Eqn. C2-5

LRFD	ASD
α = 1.0 $B_1 = \dfrac{1.0}{1-1.0(30.0\text{ kips}/371\text{ kips})} = 1.09$ M_{uy} = 1.09(12.0 kip-ft) = 13.1 kip-ft	α = 1.6 $B_1 = \dfrac{1.0}{1-1.6(20.0\text{ kips}/371\text{ kips})} = 1.09$ M_{ay} = 1.09 (8.00 kip-ft) = 8.76 kip-ft

Eqn. C2-2

Calculate the nominal bending strength about the x-x axis

Yielding limit state

$M_{nx} = M_p = F_y Z_x$ = 50 ksi(38.8 in.^{3}) = 1940 kip-in or 162 kip-ft Eqn. F2-1

Lateral-torsional buckling limit state

Since $L_p < L_b < L_r$, Equation F2-2 applies

From Manual Table 3-1, C_b = 1.14 Manual Table 3-1

$$M_{nx} = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$ Eqn. F2-2

$$M_{nx} = 1.14\left[1940\text{ kip-in.} - \left(1940\text{ kip-in.} - 0.7\left(50\text{ ksi}\right)\left(35.0\text{ in.}^3\right)\right)\left(\frac{14.0\text{ ft} - 6.85\text{ ft}}{21.8\text{ ft} - 6.85\text{ ft}}\right)\right]$$

$= 1820$ kip-in. ≤ 1940 kip-in., therefore use:

$M_{nx} = 1820$ kip-in. or 152 kip-ft **controls**

Local buckling limit state

Per Manual Table 1-1, the member is compact for $F_y = 50$ ksi, so the local buckling limit state does not apply — Manual Table 1-1

Calculate the nominal bending strength about the y-y axis — Section F6.2

Since a W10×33 has compact flanges, only the yielding limit state applies.

$M_{ny} = M_p = F_yZ_y \le 1.6F_yS_y$ — Eqn. F6-1

$= 50\text{ ksi}(14.0\text{ in.}^3) \le 1.6(50\text{ ksi})(9.20\text{ in.}^3)$

$= 700$ kip-in < 736 kip-in., therefore

Use $M_{ny} = 700$ kip-in. or 58.3 kip-ft

LRFD	ASD
$\phi_b = 0.90$ $M_{cx} = \phi_bM_{nx} = 0.90(152\text{ kip-ft}) = 137\text{ kip-ft}$ $M_{cy} = \phi_bM_{ny} = 0.90(58.3\text{ kip-ft}) = 52.5\text{ kip-ft}$	$\Omega_b = 1.67$ $M_{cx} = M_{nx}/\Omega_b = 152\text{ kip-ft}/1.67 = 91.0\text{ kip-ft}$ $M_{cy} = M_{ny}/\Omega_b = 58.3\text{ kip-ft}/1.67 = 34.9\text{ kip-ft}$

Section F2

Check limit for Equation H1-1a

LRFD	ASD
$\frac{P_r}{P_c} = \frac{P_u}{\phi_cP_n} = \frac{30.0\text{ kips}}{253\text{ kips}} = 0.119$, therefore, use Specification Equation H1.1b $\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$ $\frac{30.0\text{ kips}}{2(253\text{ kips})} + \left(\frac{91.8\text{ kip-ft}}{137\text{ kip-ft}} + \frac{13.1\text{ kip-ft}}{52.5\text{ kip-ft}}\right)$ $0.0593 + 0.920 = 0.979 \le 1.0$ **o.k**.	$\frac{P_r}{P_c} = \frac{P_a}{P_n/\Omega_c} = \frac{20.0\text{ kips}}{168\text{ kips}} = 0.119$, therefore use Specification Equation H1.1b $\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$ $\frac{20.0\text{ kips}}{2(168\text{ kips})} + \left(\frac{61.2\text{ kip-ft}}{91.0\text{ kip-ft}} + \frac{8.76\text{ kip-ft}}{34.9\text{ kip-ft}}\right)$ $0.0595 + 0.924 = 0.983 \le 1.0$ **o.k.**

Section H1.1

Eqn. H1.1b

ASDIP Steel 3.2.5

STEEL COLUMN DESIGN

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GEOMETRY

Column Designation	W10X33		
Steel Yield Strength Fy	50.0	ksi	
Modulus of Elasticity Es ...	29000	ksi	
Member Length L	14.00	ft	
Effective Length Kx-factor	1.00		
Effective Length Ky-factor	1.00		
Unbraced Length Lb	14.00	ft	OK

PROPERTIES

Area ..	9.7	in^2	Sx ...	35.0	in^3
Depth	9.7	in	Zx ...	38.8	in^3
bf	8.0	in	rx	4.19	in
tw	0.29	in	Iy	36.6	in^4
tf	0.44	in	Sy ...	9.2	in^3
k des .	0.94	in	Zy ...	14.0	in^3
Ix	171.0	in^4	ry	1.94	in

SERVICE LOADS (ASD)

Loads from a 1st-Order Analysis to be Amplified (kip, k-ft)

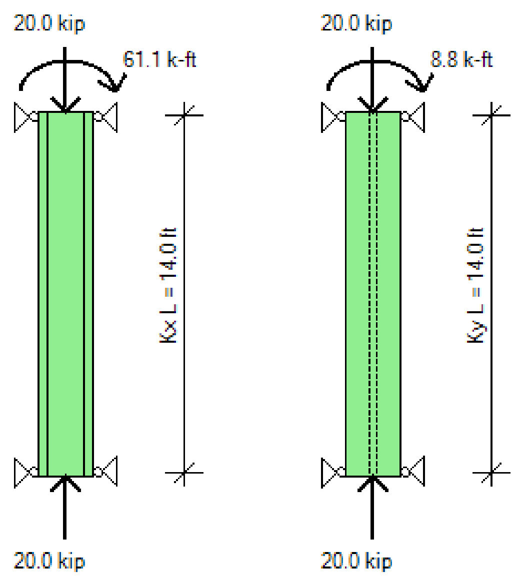
	Axial	Moment X	Moment Y
Total Axial in the Story	100		
No Lateral Translation	20.0	60.0	8.0
Lateral Translation Only ..	0.0	0.0	0.0
Amplified Loads	20.0	61.1	8.8

	X-X	Y-Y	
Story Shear	300	300	kip
Interstory Drift ..	2.30	1.50	in
M1/M2 Ratio	1.00	1.00	
Cm-factor	0.30	0.36	

COMPRESSION

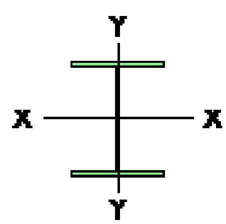
Slenderness Ratio Kx L / rx	40.1	
Slenderness Ratio Ky L / ry	86.6	
Max. Slenderness Ratio	86.6	OK

Limit States	Nominal Pn		
Flexural Buckling	280.6	kip	✓
Torsional Buckling	360.2	kip	
Flexural-Torsional Buckling	N.A.	kip	
Nominal Strength Pn	280.6	kip	
Safety Factor Ω	1.67		
Allowable Strength Pn/Ω	168.0	kip	
P / Pn/Ω Design Ratio	0.12		OK



BENDING ABOUT Y-Y

Limit States	Nominal Mn		
Yielding	58.3	k-ft	✓
Lateral-Torsional Buckling	N.A.	k-ft	
Flange Local Buckling	N.A.	k-ft	
Web Local Buckling	N.A.	k-ft	
Nominal Strength Mn	58.3	k-ft	
Safety Factor Ω	1.67		
Allowable Strength Mn/Ω ...	34.9	k-ft	
M / Mn/Ω Design Ratio	0.25		OK



BENDING ABOUT X-X

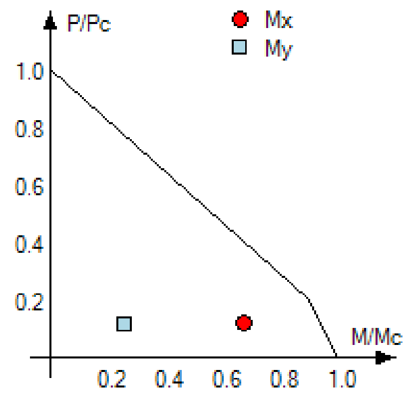
Moment at 1/4 point of Lb ...	46.0	k-ft	
Moment at 1/2 point of Lb ...	60.0	k-ft	
Moment at 3/4 point of Lb ...	46.0	k-ft	
L. T. Buckling Cb-factor	1.14		
Limit States	**Nominal Mn**		
Yielding	161.7	k-ft	
Lateral-Torsional Buckling	152.2	k-ft	✓
Flange Local Buckling	N.A.	k-ft	
Web Local Buckling	N.A.	k-ft	
Nominal Strength Mn	152.2	k-ft	
Safety Factor Ω	1.67		
Allowable Strength Mn/Ω ...	91.1	k-ft	
M / Mn/Ω Design Ratio	0.67		OK

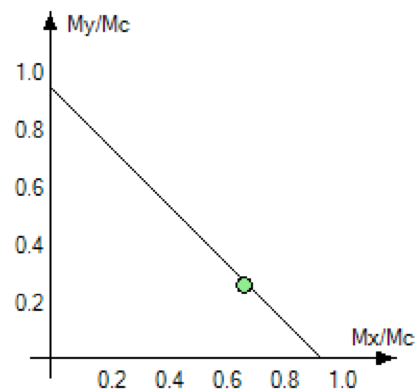
COMBINED FORCES

AISC Equation {H1-1a}	N.A.	
AISC Equation {H1-1b}	0.98	OK

LOCAL BUCKLING

Flanges in Flexure	Compact
Flanges in Compression	Non-slender
Web in Flexure	Compact
Web in Compression	Non-slender





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GEOMETRY

Column Designation	W10X33		
Steel Yield Strength Fy	50.0	ksi	
Modulus of Elasticity Es	29000	ksi	
Member Length L	14.00	ft	
Effective Length Kx-factor ..	1.00		
Effective Length Ky-factor ..	1.00		
Unbraced Length Lb	14.00	ft	OK

PROPERTIES

Area ..	9.7	in^2	Sx ...	35.0	in^3
Depth	9.7	in	Zx ...	38.8	in^3
bf	8.0	in	rx	4.19	in
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tf	0.44	in	Sy ...	9.2	in^3
k des .	0.94	in	Zy ...	14.0	in^3
Ix	171.0	in^4	ry	1.94	in

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Loads from a 1st-Order Analysis to be Amplified (kip, k-ft)

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Lateral Translation Only	0.0	0.0	0.0
Amplified Loads	20.0	61.1	8.8

	X-X	Y-Y	
Story Shear	300	300	kip
Interstory Drift	2.30	1.50	in
M1/M2 Ratio	1.00	1.00	
Cm-factor	0.30	0.36	

DESIGN FOR COMPRESSION

Slender unstiffened Qs = 1.00 , Slender stiffened Qa = 1.00 , Q = Qs * Qa = 1.00 — AISC E7

Slenderness ratio *Kx L / rx* = 1.00 * 14.00 / 4.19 = 40.1

Slenderness ratio *Ky L / ry* = 1.00 * 14.00 / 1.94 = 86.6

Maximum slenderness ratio = Max (40.1, 86.6) = 86.6 — AISC E3

Elastic buckling $Fe = \dfrac{\pi^2 * E}{(K\,L\,/\,r)^2} = \dfrac{3.14^2 * 29000}{86.6^2} = 38.2$ ksi — AISC Eq. E3-4

*Fe >= 0.44 * Q * Fy* (38.2 >= 0.44 * 1.00 * 50.0 = 22.0)

Flexural buckling stress $Fcr = Q * Fy * (0.658)^{Q * Fy / Fe}$ — AISC Eq. E3-2

$Fcr = 1.00 * 50.0 * (0.658)^{1.00 * 50.0 / 38.2} = 28.9$ ksi

$$Fe = \left[\frac{\pi^2 * E * Cw}{(Ky\,L)^2} + G * J\right] \frac{1}{Ix + Iy}$$ AISC Eq. E4-4

$$= \left[\frac{\pi^2 * 29000 * 791.0}{(1.00 * 14.0 * 12)^2} + 11200 * 0.6\right] \frac{1}{171.0 + 36.6} = 70.1 \text{ ksi}$$

*Fe >= 0.44 * Q * Fy* (70.1 >= 0.44 * 1.00 * 50.0 = 22.0)

Torsional buckling $Fcr = Q * Fy * (0.658)^{Q * Fy / Fe}$ — AISC Eq. E3-2

$Fcr = 1.00 * 50.0 * (0.658)^{1.00 * 50.0 / 70.1} = 37.1$ ksi

Compressive strength *Pn = Fcr * Ag* = 28.9 * 9.7 = 280.6 kip — AISC Eq. E3-1

Controlling limit state: Flexural Buckling

Compressive design ratio = $\dfrac{P}{Pn / \Omega} = \dfrac{20.0}{280.6 / 1.67} = 0.12$ < 1.0 OK — AISC E1

DESIGN FOR FLEXURE

*Cb = 12.5 Mmax * Rm / (2.5 Mmax + 3 Ma + 4 Mb + 3 Mc)* — AISC Eq F1-1

= 12.5 * 61.1 * 1.0 / (2.5 * 61.1 + 3 * 46.0 + 4 * 60.0 + 3 * 46.0 = 1.14

- Yielding

Plastic moment *Mpx = Fy * Zx* = 50.0 * 38.8 = 1940.0 k-in — AISC Eq. F2-1

Nominal strength *Mnx = Mpx* = 1940.0 / 12 = 161.7 k-ft

- Lateral-Torsional Buckling

$$Lp = 1.76 \cdot ry \cdot \sqrt{\frac{E}{Fy}} = 1.76 \cdot 1.9 \cdot \sqrt{\frac{29000}{50}}$$ Lp = 82.2 in — AISC Eq. F2-5

$$rts = \sqrt{\frac{\sqrt{Iy \cdot Cw}}{Sx}} = \sqrt{\frac{\sqrt{36.6 \cdot 791}}{35}}$$ rts = 2.2 in — AISC Eq. F2-7

ho = d - tf = 9.7 - 0.4 = 9.3 in

c = 1.0 (doubly symmetric I-shape) — AISC Eq. F2-8a

$$Lr = \frac{1.95 \cdot rts \cdot E}{(0.7 \cdot Fy)} \cdot \sqrt{\frac{J \cdot c}{(Sx \cdot ho)} + \sqrt{\left(\frac{J \cdot c}{(Sx \cdot ho)}\right)^2 + 6.76 \cdot \left(\frac{0.7 \cdot Fy}{E}\right)^2}}$$ — AISC Eq. F2-6

$$\frac{1.95 \cdot 2.2 \cdot 29000}{0.7 \cdot 50} \cdot \sqrt{\frac{0.6 \cdot 1}{35 \cdot 9.3} + \sqrt{\left(\frac{0.6 \cdot 1}{35 \cdot 9.3}\right)^2 + 6.76 \cdot \left(\frac{0.7 \cdot 50}{29000}\right)^2}}$$ Lr = 261.9 in

$$Fcr = \frac{Cb * \pi^2 * E}{(Lb / rts)^2} \sqrt{1 + \frac{0.078 \cdot J \cdot c}{(Sx \cdot ho)} \cdot \left(\frac{Lb}{rts}\right)^2}$$ — AISC Eq. F2-4

$$= \frac{1.14 * \pi^2 * 29000.0}{(14.0 * 12 / 2.2)^2} \sqrt{1 + \frac{0.078 \cdot 0.6 \cdot 1}{35 \cdot 9.3} \cdot \left(\frac{14}{2.2}\right)^2}$$ Fcr = 75.8 ksi

Nominal strength *Mnx = Cb * [Mpx - (Mpx - 0.7 * Fy * Sx) * (Lb - Lp) / (Lr - Lp)]* — AISC Eq. F2-2

Mnx = 1.14 * [1940.0 - (1940.0 - 0.7 * 50.0 * 35.0)(14.0 * 12 - 82.2) / (261.9 - 82.2)] / 12 = 152.2 k-ft

X - Controlling limit state: Lateral-Torsional Buckling

X - Flexural design ratio = $\frac{Mx}{Mnx / \Omega} = \frac{61.1}{152.2 / 1.67}$ = 0.67 < 1.0 OK — AISC F1

- Yielding

*Mpy = Min (Fy * Zy, 1.6 * Fy * Sy)* =Min (50.0 * 14.0, 1.6 * 50.0 * 9.2) = 700.0 k-in

Nominal strength *Mny = Mpy* = 700.0 / 12 = 58.3 k-ft — AISC Eq. F6-1

- Flange Local Buckling

Nominal strength *Mny = N.A.* (compact flanges)

Y - Controlling limit state: Yielding

Y - Flexural design ratio = $\frac{My}{Mny / \Omega} = \frac{8.8}{58.3 / 1.67}$ = 0.25 < 1.0 OK — AISC F1

STEEL COLUMN DESIGN

DESIGN FOR COMBINED FORCES

Allowable axial strength $Pc = \dfrac{Pn}{\Omega} = \dfrac{280.6}{1.67} = 168.0$ kip — AISC E1

Allowable flexural strength $Mcx = \dfrac{Mnx}{\Omega} = \dfrac{152.2}{1.67} = 91.1$ kip — AISC F1

Allowable flexural strength $Mcy = \dfrac{Mny}{\Omega} = \dfrac{58.3}{1.67} = 34.9$ kip — AISC F1

Combined forces ratio $= \dfrac{P}{2\,Pc} + \left[\dfrac{Mx}{Mcx} + \dfrac{My}{Mcy}\right]$ — AISC Eq. H1-1b

$= \dfrac{20.0}{2 * 168.0} + \left[\dfrac{61.1}{91.1} + \dfrac{8.8}{34.9}\right] = 0.98 \quad < 1.0$ OK